

# ANALYSIS OF THE $B \rightarrow K_2^*(1430), a_2(1320), f_2(1270)$ FORM-FACTORS WITH LIGHT-CONE QCD SUM RULES

Zhi-Gang Wang<sup>1</sup>

Department of Physics, North China Electric Power University, Baoding 071003, P. R. China

## Abstract

In this article, we study the  $B \rightarrow K_2^*(1430), a_2(1320), f_2(1270)$  form-factors with the light-cone QCD sum rules, where the  $B$ -meson light-cone distribution amplitudes are used. In calculations, we observe that the line-shapes of the  $B$ -meson light-cone distribution amplitude  $\phi_+(\omega)$  have significant impacts on the values of the form-factors, and expect to obtain severe constraints on the parameters of the  $B$ -meson light-cone distribution amplitudes from the experimental data in the future.

PACS numbers: 12.38.Lg; 13.20.He

**Key Words:**  $B$  meson, Light-cone QCD sum rules

## 1 Introduction

The semi-leptonic and radiative  $B$ -decays to the light tensor mesons play an important role in testing the standard model and searching for new physics. At the quark level, the transitions  $b \rightarrow s(d)\gamma$  and  $b \rightarrow s(d)\ell^+\ell^-$  occur through the flavor changing neutral currents, which are forbidden at the tree-level in the standard model, provide fertile ground for testing the standard model at the loop-level as well as obtaining useful information about new physics effects. The  $B \rightarrow T$  form-factors enter the semi-leptonic decays and radiative decays, and serve as the basic parameters, their values with improved precision can reduce the theoretical uncertainties. Experimentally, the radiative decays  $B^0 \rightarrow K_2^{*0}(1430)\gamma$  and  $B^+ \rightarrow K_2^{*+}(1430)\gamma$  have been observed in the past years [1, 2]. Other radiative decays and semi-leptonic decays are expected to be observed at the KEK-B and the LHCb in the future. The tensor mesons cannot be produced from the local  $V \pm A$  currents of the standard model, the  $B \rightarrow T$  form-factors also play an important role in studying the two-body hadronic decays  $B \rightarrow TM$  (with  $M = P, V, A$ ), the calculations based on the naive factorization approach and QCD-improved factorization approach cannot account for the experimental data satisfactorily [3], the non-factorizable contributions maybe large enough. Comparing with the two-body hadronic  $B$ -decays, the semi-leptonic and radiative  $B$ -decays suffer from much less theoretical uncertainties involving the hadronic matrix elements.

The  $B \rightarrow T$  form-factors concern the nonperturbative sector of QCD and have been calculated in the ISGW model [4] and its improved version (the ISGW2 model) [5], the covariant light-front quark model [6], the light-cone sum rules approach (where the light-cone distribution amplitudes of the tensor mesons are used) [7], the large energy effective theory [8, 9, 10] and the perturbative QCD approach [11], etc. Another calculation based on the light-cone QCD sum rules is useful.

In Refs.[12, 13], Khodjamirian et al obtain new sum rules (the so-called  $B$ -meson light-cone QCD sum rules) from the correlation functions expanded near the light-cone in terms of the  $B$ -meson distribution amplitudes for the  $B \rightarrow \pi, K, \rho, K^*$  form-factors. Furthermore, they suggest QCD sum rules motivated models for the three-particle  $B$ -meson light-cone distribution amplitudes, which satisfy the relations between the two-particle and three-particle  $B$ -meson light-cone distribution amplitudes derived from the QCD equations of motion and heavy-quark symmetry [14]. The new QCD sum rules have been applied successfully to calculate the  $B \rightarrow a_1(1260), D, D^*$  form-factors, etc [15, 16]. In this article, we use the  $B$ -meson light-cone QCD sum rules to study the  $B \rightarrow K_2^*(1430), a_2(1320), f_2(1270)$  form-factors.

The  $B$ -meson light-cone distribution amplitudes play an important role in the exclusive  $B$ -decays, the inverse moment of the two-particle light-cone distribution amplitude  $\phi_+(\omega)$  enters

<sup>1</sup> E-mail, wangzgyiti@yahoo.com.cn.

many factorization formulas [17, 18]. The two-particle  $B$ -meson light-cone distribution amplitudes have been studied with the QCD sum rules and renormalization group equation [12, 19, 20, 21, 22, 23, 24, 25]. Although the QCD sum rules cannot be used for a direct calculation of the distribution amplitudes, it can provide constraints which have to be implemented within the QCD motivated models [22]. Comparing with the light pseudoscalar mesons and vector mesons, the  $B$ -meson light-cone distribution amplitudes have received relatively little attention. Our knowledge about the nonperturbative parameters which determine those light-cone distribution amplitudes is limited and an additional application (or estimation) based on the light-cone QCD sum rules is useful.

On the other hand, if those form-factors are extracted from the experimental data on the semi-leptonic decays and radiative decays at the KEK-B and LHCb in the future, we get feedback, and obtain severe constraints on the input parameters of the  $B$ -meson light-cone distribution amplitudes. At the LHCb, the  $b\bar{b}$  pairs will be copiously produced with the cross section about  $500 \mu b$ .

The article is arranged as: in Sect.2, we derive the  $B \rightarrow K_2^*(1430)$ ,  $a_2(1320)$ ,  $f_2(1270)$  form-factors with the light-cone QCD sum rules; in Sect.3, the numerical result and discussion; and Sect.4 is reserved for our conclusion.

## 2 $B \rightarrow K_2^*(1430)$ , $a_2(1320)$ , $f_2(1270)$ form-factors with light-cone QCD sum rules

In the following, we write down the definitions for the  $B \rightarrow K_2^*(1430)$  form-factors  $V(q^2)$ ,  $A_1(q^2)$ ,  $A_2(q^2)$ ,  $A_3(q^2)$ ,  $A_0(q^2)$ ,  $T_1(q^2)$ ,  $T_2(q^2)$ , and  $T_3(q^2)$  [11, 26, 27],

$$\begin{aligned}
\langle K_2^*(p) | \bar{s}(0) \gamma^\mu b(0) | B(P) \rangle &= -\frac{2}{m_B + m_{K_2^*}} \epsilon^{\mu\lambda\tau\rho} \epsilon_\lambda^* P_\tau p_\rho V(q^2), \\
\langle K_2^*(p) | \bar{s}(0) \gamma_\mu \gamma_5 b(0) | B(P) \rangle &= i \left\{ (m_B + m_{K_2^*}) \epsilon_\mu^* A_1(q^2) - \frac{\epsilon^* \cdot P}{m_B + m_{K_2^*}} (P + p)_\mu A_2(q^2) \right. \\
&\quad \left. - 2m_{K_2^*} \frac{\epsilon^* \cdot P}{q^2} q_\mu [A_3(q^2) - A_0(q^2)] \right\}, \\
\langle K_2^*(p) | \bar{s}(0) \sigma^{\mu\nu} q_\nu b(0) | B(P) \rangle &= 2i \epsilon^{\mu\lambda\rho\tau} P_\lambda p_\rho \epsilon_\tau^* T_1(q^2), \\
\langle K_2^*(p) | \bar{s}(0) \sigma_{\mu\nu} \gamma_5 q^\nu b(0) | B(P) \rangle &= T_2(q^2) \left[ (m_B^2 - m_{K_2^*}^2) \epsilon_\mu^* - \epsilon^* \cdot P (P + p)_\mu \right] \\
&\quad + T_3(q^2) \epsilon^* \cdot P \left[ q_\mu - \frac{q^2}{m_B^2 - m_{K_2^*}^2} (P + p)_\mu \right], \tag{1}
\end{aligned}$$

where

$$A_3(q^2) = \frac{m_B + m_{K_2^*}}{2m_{K_2^*}} A_1(q^2) - \frac{m_B - m_{K_2^*}}{2m_{K_2^*}} A_2(q^2), \tag{2}$$

$A_0(0) = A_3(0)$ ,  $\epsilon^\alpha = \frac{\epsilon^{\alpha\beta} q_\beta}{m_B}$ , and the  $\epsilon^{\alpha\beta}$  is the polarization vector of the tensor meson  $K_2^*(1430)$ . In this article, we write down the calculations for the  $B \rightarrow K_2^*(1430)$  form-factors explicitly, and obtain others using the flavor  $SU(3)$  symmetry.

We study the form-factors with the two-point correlation functions  $\Pi_{\alpha\beta\mu}^k(p, q)$ ,

$$\begin{aligned}
\Pi_{\alpha\beta\mu}^k(p, q) &= i \int d^4x e^{ip \cdot x} \langle 0 | T \{ J_{\alpha\beta}(x) J_{\mu}^k(0) \} | B(P) \rangle, \\
J_{\alpha\beta}(x) &= \frac{i}{2} \left\{ \bar{u}(x) \gamma_{\alpha} \left[ \vec{D}_{\beta}(x) - \overleftarrow{D}_{\beta}(x) \right] s(x) + \bar{u}(x) \gamma_{\beta} \left[ \vec{D}_{\alpha}(x) - \overleftarrow{D}_{\alpha}(x) \right] s(x) \right\}, \\
J_{\mu}^1(x) &= \bar{s}(x) \gamma_{\mu} b(x), \\
J_{\mu}^2(x) &= \bar{s}(x) \gamma_{\mu} \gamma_5 b(x), \\
J_{\mu}^3(x) &= \bar{s}(x) \sigma_{\mu\nu} q^{\nu} b(x), \\
J_{\mu}^4(x) &= \bar{s}(x) \sigma_{\mu\nu} \gamma_5 q^{\nu} b(x),
\end{aligned} \tag{3}$$

where  $\vec{D}_{\alpha}(x) = \vec{\partial}_{\alpha}(x) - ig_s A_{\alpha}(x)$ ,  $\overleftarrow{D}_{\alpha}(x) = \overleftarrow{\partial}_{\alpha}(x) + ig_s A_{\alpha}(x)$ ,  $A_{\alpha} = \frac{\lambda^a}{2} A_{\alpha}^a$ , and  $k = 1, 2, 3, 4$ .

According to the quark-hadron duality [28, 29], we can insert a complete set of intermediate states with the same quantum numbers as the current operator  $J_{\alpha\beta}(x)$  into the correlation functions  $\Pi_{\alpha\beta\mu}^k(p, q)$  to obtain the hadronic representation. After isolating the ground state contributions from the pole term of the tensor meson  $K_2^*(1430)$ , we obtain the results,

$$\Pi_{\alpha\beta\mu}^1(p, q) = \frac{f_{K_2^*} m_{K_2^*}^2 m_B V(q^2)}{(m_B + m_{K_2^*}) (m_{K_2^*}^2 - p^2)} (-v_{\alpha} \varepsilon_{\beta\mu\lambda\tau} p^{\lambda} v^{\tau} - v_{\beta} \varepsilon_{\alpha\mu\lambda\tau} p^{\lambda} v^{\tau}) + \dots, \tag{5}$$

$$\begin{aligned}
Z^{\alpha} Z^{\beta} \Pi_{\alpha\beta\mu}^2(p, q) &= \frac{if_{K_2^*} m_{K_2^*}^2}{m_{K_2^*}^2 - p^2} \left[ (m_B + m_{K_2^*}) A_1(q^2) Z_{\mu} Z \cdot v - \frac{m_B^2 \tilde{A}_2(q^2)}{m_B + m_{K_2^*}} (Z \cdot v)^2 v_{\mu} \right. \\
&\quad \left. - \frac{m_B \tilde{A}_3(q^2)}{m_B + m_{K_2^*}} (Z \cdot v)^2 p_{\mu} \right] + \dots,
\end{aligned} \tag{6}$$

$$\Pi_{\alpha\beta\mu}^3(p, q) = \frac{if_{K_2^*} m_{K_2^*}^2 m_B T_1(q^2)}{m_{K_2^*}^2 - p^2} (-v_{\alpha} \varepsilon_{\beta\mu\lambda\tau} p^{\lambda} v^{\tau} - v_{\beta} \varepsilon_{\alpha\mu\lambda\tau} p^{\lambda} v^{\tau}) + \dots, \tag{7}$$

$$Z^{\alpha} Z^{\beta} \Pi_{\alpha\beta\mu}^4(p, q) = \frac{f_{K_2^*} m_{K_2^*}^2}{m_{K_2^*}^2 - p^2} \left[ (m_B^2 - m_{K_2^*}^2) T_2(q^2) Z_{\mu} Z \cdot v - m_B^2 \tilde{T}_3(q^2) (Z \cdot v)^2 v_{\mu} \right] + \dots \tag{8}$$

where

$$\begin{aligned}
\tilde{A}_2(q^2) &= A_2(q^2) + \frac{2m_{K_2^*} (m_B + m_{K_2^*})}{q^2} [A_3(q^2) - A_0(q^2)], \\
\tilde{A}_3(q^2) &= A_2(q^2) - \frac{2m_{K_2^*} (m_B + m_{K_2^*})}{q^2} [A_3(q^2) - A_0(q^2)], \\
\tilde{T}_3(q^2) &= T_2(q^2) - T_3(q^2) \left( 1 - \frac{q^2}{m_B^2 - m_{K_2^*}^2} \right),
\end{aligned} \tag{9}$$

the  $Z_{\mu}$  is a four-vector with  $Z^2 = 1$ ,  $P_{\alpha} = p_{\alpha} + q_{\alpha} = m_B v_{\alpha}$ , and the decay constant (or pole residue)  $f_{K_2^*}$  is defined by

$$\begin{aligned}
\langle 0 | J_{\alpha\beta}(0) | K_2^*(p) \rangle &= f_{K_2^*} m_{K_2^*}^2 \epsilon_{\alpha\beta}, \\
\sum_{\lambda} \epsilon_{\alpha\beta}^*(\lambda, p) \epsilon_{\mu\nu}(\lambda, p) &= \frac{T_{\alpha\mu} T_{\beta\nu} + T_{\alpha\nu} T_{\beta\mu}}{2} - \frac{T_{\alpha\beta} T_{\mu\nu}}{3}, \\
T_{\alpha\beta} &= -g_{\alpha\beta} + \frac{p_{\alpha} p_{\beta}}{p^2}.
\end{aligned} \tag{10}$$

The tensor current  $J_{\alpha\beta}(x)$  maybe also have non-vanishing coupling with the vector meson  $K^*$ ,

$$\langle 0 | J_{\alpha\beta}(0) | K^*(p) \rangle = g_{K^*} m_{K^*}^2 (\eta_{\alpha} p_{\beta} + \eta_{\beta} p_{\alpha}), \tag{11}$$

where the  $\eta_\alpha$  is the polarization vector and the  $g_{K^*}$  is the decay constant (or pole residue). In this article, we derive the QCD sum rules with the tensor structures  $v_\alpha \varepsilon_{\beta\mu\lambda\tau} p^\lambda v^\tau + v_\beta \varepsilon_{\alpha\mu\lambda\tau} p^\lambda v^\tau$ ,  $Z_\mu Z \cdot v$ ,  $(Z \cdot v)^2 v_\mu$  and  $(Z \cdot v)^2 p_\mu$  to avoid possible contaminations from the  $K^*$  meson.

In the following, we briefly outline the operator product expansion for the correlation functions  $\Pi_{\alpha\beta\mu}^k(p, q)$  in perturbative QCD. The calculations are performed at the large space-like momentum region  $p^2 \ll 0$  and  $0 \leq q^2 < m_b^2 + \frac{m_b p^2}{\bar{\Lambda}}$ ,  $\bar{\Lambda} = m_B - m_b$ . We write down the propagator of a massive quark in the external gluon field in the Fock-Schwinger gauge and the  $B$ -meson light-cone distribution amplitudes firstly [13, 19, 30],

$$S_{ij}(x, y) = i \int \frac{d^4 k}{(2\pi)^4} e^{-ik(x-y)} \left\{ \frac{k + m}{k^2 - m^2} \delta_{ij} - \int_0^1 du g_s G_{ij}^{\mu\nu}(ux + (1-u)y) \left[ \frac{1}{2} \frac{k + m}{(k^2 - m^2)^2} \sigma_{\mu\nu} - \frac{1}{k^2 - m^2} u(x-y)_\mu \gamma_\nu \right] \right\}, \quad (12)$$

$$\langle 0 | \bar{q}_\alpha(x) h_{v\beta}(0) | B(v) \rangle = -\frac{if_B m_B}{4} \int_0^\infty d\omega e^{-i\omega v \cdot x} \left\{ (1 + \not{v}) \left[ \phi_+(\omega) - \frac{\phi_+(\omega) - \phi_-(\omega)}{2v \cdot x} \not{x} \right] \gamma_5 \right\}_{\beta\alpha},$$

$$\begin{aligned} \langle 0 | \bar{q}_\alpha(x) G_{\lambda\rho}(ux) h_{v\beta}(0) | B(v) \rangle &= \frac{f_B m_B}{4} \int_0^\infty d\omega \int_0^\infty d\xi e^{-i(\omega+u\xi)v \cdot x} \left\{ (1 + \not{v}) \left[ (v_\lambda \gamma_\rho - v_\rho \gamma_\lambda) \right. \right. \\ &\quad \left. \left( \Psi_A(\omega, \xi) - \Psi_V(\omega, \xi) \right) - i\sigma_{\lambda\rho} \Psi_V(\omega, \xi) - \frac{x_\lambda v_\rho - x_\rho v_\lambda}{v \cdot x} X_A(\omega, \xi) \right. \\ &\quad \left. \left. + \frac{x_\lambda \gamma_\rho - x_\rho \gamma_\lambda}{v \cdot x} Y_A(\omega, \xi) \right] \gamma_5 \right\}_{\beta\alpha}, \end{aligned} \quad (13)$$

where

$$\begin{aligned} \phi_+(\omega) &= \frac{\omega}{\omega_0^2} e^{-\frac{\omega}{\omega_0}}, \quad \phi_-(\omega) = \frac{1}{\omega_0} e^{-\frac{\omega}{\omega_0}}, \\ \Psi_A(\omega, \xi) &= \Psi_V(\omega, \xi) = \frac{\lambda_E^2}{6\omega_0^4} \xi^2 e^{-\frac{\omega+\xi}{\omega_0}}, \\ X_A(\omega, \xi) &= \frac{\lambda_E^2}{6\omega_0^4} \xi(2\omega - \xi) e^{-\frac{\omega+\xi}{\omega_0}}, \\ Y_A(\omega, \xi) &= -\frac{\lambda_E^2}{24\omega_0^4} \xi(7\omega_0 - 13\omega + 3\xi) e^{-\frac{\omega+\xi}{\omega_0}}, \end{aligned} \quad (14)$$

the  $\omega_0$  and  $\lambda_E^2$  are parameters of the  $B$ -meson light-cone distribution amplitudes. In this article, we take the two-particle and three-particle  $B$ -meson light-cone distribution amplitudes suggested in Ref.[19] and Ref.[13], respectively. They obey the powerful constraints derived in Ref.[14] and the relations between the matrix elements of the local operators and the moments of the light-cone distribution amplitudes, if the conditions  $\omega_0 = \frac{2}{3}\bar{\Lambda}$  and  $\lambda_E^2 = \lambda_H^2 = \frac{3}{2}\omega_0^2 = \frac{2}{3}\bar{\Lambda}^2$  are satisfied [19].

We contract the quark fields with Wick theorem, substitute the  $s$  quark propagator and the  $B$ -meson light-cone distribution amplitudes into the correlation functions  $\Pi_{\alpha\beta\mu}^k(p, q)$ , then complete the integrals over the variables  $x$  and  $k$ , finally obtain the representation at the level of quark-gluon degrees of freedom. In calculations, we take into account the contributions from the two-particle and three-particle  $B$ -meson light-cone distribution amplitudes, the gluons presented in the covariant derivatives  $\vec{D}_\alpha$ ,  $\overleftarrow{D}_\alpha$  (or emitted from the vertex) and emitted from the intermediate

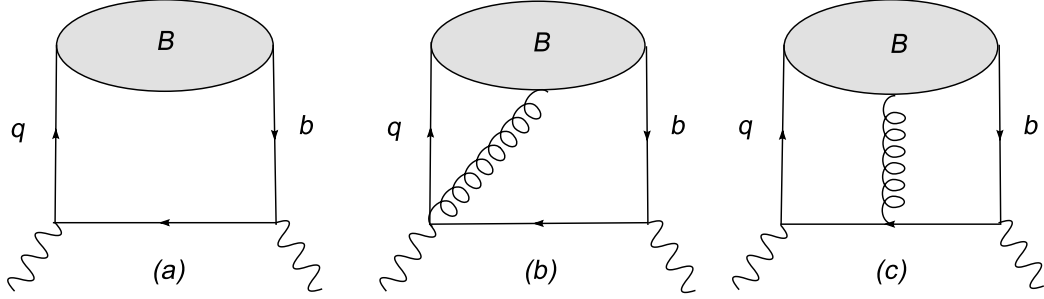


Figure 1: The diagrams contribute to the form-factors in the operator product expansion.

quark lines, which correspond to the diagrams (b) and (c) respectively in Fig.1, both contribute to the three-particle  $B$ -meson light-cone distribution amplitudes. In calculating the diagrams (b-c) in Fig.1, we use the Fock-Schwinger gauge  $x^\mu A_\mu^a(x) = 0$  to express the gluon field  $A_\mu$  in terms of the gluon field strength tensor  $G_{\mu\nu}$ ,  $A_\mu(x) = \int_0^1 d\tau \tau x_\nu G_{\nu\mu}(\tau x)$ , and then extract the three-particle  $B$ -meson light-cone distribution amplitudes.

In the region of small  $\omega$ , the exponential form of (or the Gaussian-like) distribution amplitude  $\phi_+(\omega)$  is numerically close to the more elaborated model (the Braun-Ivanov-Korchensky model, or the BIK model) suggested in Ref.[22],

$$\phi_+(\omega, \mu = 1\text{GeV}) = \frac{4\omega}{\pi\lambda_B(1+\omega^2)} \left[ \frac{1}{1+\omega^2} - 2\frac{\sigma_B-1}{\pi^2} \ln \omega \right], \quad (15)$$

where  $\omega_0 = \lambda_B$ , and  $\omega$  is in unit of GeV. The parameters  $\lambda_B$  and  $\sigma_B$  are determined by the QCD sum rules including the radiative and nonperturbative corrections in the heavy quark effective theory. There are other phenomenological models for the two-particle  $B$ -meson light-cone distribution amplitudes, for example, the  $k_T$  factorization formalism [31, 32]. In this article, we use the QCD sum rules motivated models.

After matching with the hadronic representation below the continuum thresholds  $s_0$ , we obtain the sum rules for the form-factors  $V(q^2)$ ,  $A_1(q^2)$ ,  $\tilde{A}_2(q^2)$ ,  $\tilde{A}_3(q^2)$ ,  $T_1(q^2)$ ,  $T_2(q^2)$  and  $\tilde{T}_3(q^2)$ , respectively,

$$\begin{aligned} V(q^2) = & \frac{f_B m_B (m_B + m_{K_2^*})}{f_{K_2^*} m_{K_2^*}^2} e^{\frac{m_{K_2^*}^2}{M^2}} \left\{ \int_0^{\sigma_0} d\sigma \omega' \left\{ \frac{\phi_+(\omega')}{\bar{\sigma}} - \frac{m_s}{\bar{\sigma}^2 M^2} [\tilde{\phi}_+(\omega') - \tilde{\phi}_-(\omega')] \right\} e^{-\frac{s}{M^2}} \right. \\ & + \int_0^{\sigma_0} d\sigma \left\{ \frac{[2um_s + (2u-1)\omega] [\Psi_A(\omega, \xi) - \Psi_V(\omega, \xi)]}{\bar{\sigma}^2 M^2} - \frac{2[u\tilde{\omega} + (1-u)\omega] \Psi_V(\omega, \xi)}{\bar{\sigma}^2 M^2} \right. \\ & + \frac{2u(\tilde{\omega} + \omega) X_A(\omega, \xi)}{\bar{\sigma}^2 M^2} + \frac{2m_s(\omega - 2u\tilde{\omega}) \tilde{Y}(\omega, \xi)}{\bar{\sigma}^3 M^4} + \frac{\tilde{X}_A(\omega, \xi)}{\bar{\sigma}^2 M^2} \left[ \frac{1+8u}{2} \right. \\ & \left. \left. + \frac{2(2um_s^2 - m_s\omega + \omega\tilde{\omega})}{\bar{\sigma} M^2} + \frac{2\omega}{m_B \bar{\sigma}} \left( 1 - \frac{\tilde{m}_B^2}{2M^2} \right) \right] \right\} e^{-\frac{s}{M^2}} \right\}, \quad (16) \end{aligned}$$

$$\begin{aligned}
A_1(q^2) = & \frac{2f_B m_B^2}{f_{K_2^*} m_{K_2^*}^2 (m_B + m_{K_2^*})} e^{\frac{m_{K_2^*}^2}{M^2}} \left\{ \int_0^{\sigma_0} d\sigma \omega' \left\{ \frac{\phi_+(\omega')}{\bar{\sigma}} \left( m_s - \omega' + \frac{\tilde{m}_B^2}{2m_B} \right) \right. \right. \\
& - \left[ \tilde{\phi}_+(\omega') - \tilde{\phi}_-(\omega') \right] \frac{m_s}{\bar{\sigma}^2} \left[ \frac{m_s - \omega'}{M^2} - \frac{1}{2m_B} \left( 1 - \frac{\tilde{m}_B^2}{M^2} \right) \right] \left. \right\} e^{-\frac{s}{M^2}} \\
& + \int_0^{\sigma_0} \tilde{d\sigma} \left\{ [\Psi_A(\omega, \xi) - \Psi_V(\omega, \xi)] \left[ -\frac{u}{\bar{\sigma}} + \frac{2um_s(m_s - \tilde{\omega}) + \omega(m_s + \tilde{\omega}) - 2u\omega\tilde{\omega}}{\bar{\sigma}^2 M^2} \right. \right. \\
& - \frac{2u(m_s + \omega) - \omega}{2m_B \bar{\sigma}^2} \left( 1 - \frac{\tilde{m}_B^2}{M^2} \right) \left. \right] + \frac{\Psi_V(\omega, \xi)}{\bar{\sigma}^2} \left[ \frac{2[u\tilde{\omega}(\tilde{\omega} - m_s) + (1-u)\omega\tilde{\omega}]}{M^2} \right. \\
& + \frac{u(\tilde{\omega} - \omega) + \omega}{m_B} \left( 1 - \frac{\tilde{m}_B^2}{M^2} \right) \left. \right] + \frac{2uX_A(\omega, \xi)}{\bar{\sigma}^2} \left[ \frac{\tilde{\omega}(m_s - \tilde{\omega} - \omega)}{M^2} - \frac{\omega + \tilde{\omega}}{2m_B} \left( 1 - \frac{\tilde{m}_B^2}{M^2} \right) \right] \\
& + \frac{\tilde{X}_A(\omega, \xi)}{\bar{\sigma}^2} \left[ \frac{m_s + 6\omega + (1-12u)\tilde{\omega}}{2M^2} - \frac{1+8u}{4m_B} \left( 1 - \frac{\tilde{m}_B^2}{M^2} \right) + \frac{4um_s^2(m_s - \tilde{\omega} - \omega)}{\bar{\sigma} M^4} \right. \\
& - \frac{4um_s^2}{\bar{\sigma} m_B M^2} \left( 1 - \frac{\tilde{m}_B^2}{2M^2} \right) - \frac{4\omega}{\bar{\sigma} M^2} \left( 1 - \frac{s}{2M^2} \right) - \frac{2\omega}{\bar{\sigma} m_B^2} \left( \frac{1}{2} - \frac{\tilde{m}_B^2}{M^2} + \frac{\tilde{m}_B^4}{4M^4} \right) \left. \right] \\
& + \frac{2\tilde{Y}_A(\omega, \xi)}{\bar{\sigma}^2 M^2} \left[ (2u-1)(\tilde{\omega} + \omega) + 2u\tilde{\omega} + \frac{m_s[2u\tilde{\omega}(\tilde{\omega} - m_s) - \omega(\tilde{\omega} + m_s) + 2um_s\omega]}{\bar{\sigma} M^2} \right. \\
& \left. \left. + \frac{m_s(2u\tilde{\omega} - \omega)}{\bar{\sigma} m_B} \left( 1 - \frac{\tilde{m}_B^2}{2M^2} \right) \right] \right\} e^{-\frac{s}{M^2}} \Bigg\}, \tag{17}
\end{aligned}$$

$$\begin{aligned}
\tilde{A}_2(q^2) = & -\frac{2f_B(m_B + m_{K_2^*})}{f_{K_2^*} m_{K_2^*}^2} e^{\frac{m_{K_2^*}^2}{M^2}} \left\{ \int_0^{\sigma_0} d\sigma \omega'^2 \left\{ \frac{2\phi_+(\omega')}{\bar{\sigma}} + \frac{m_s}{\bar{\sigma}^2 M^2} [\tilde{\phi}_+(\omega') - \tilde{\phi}_-(\omega')] \right\} e^{-\frac{s}{M^2}} \right. \\
& + \int_0^{\sigma_0} \tilde{d\sigma} \left\{ \frac{2[(1-2u)\omega\tilde{\omega} - 2m_s\omega - 2u\tilde{\omega}^2][\Psi_A(\omega, \xi) - \Psi_V(\omega, \xi)]}{\bar{\sigma}^2 M^2} \right. \\
& + \frac{4\tilde{\omega}[(1-u)\omega - u\tilde{\omega}]\Psi_V(\omega, \xi)}{\bar{\sigma}^2 M^2} + \frac{4u\tilde{\omega}(\omega + \tilde{\omega})X_A(\omega, \xi)}{\bar{\sigma}^2 M^2} \\
& + \frac{\tilde{X}_A(\omega, \xi)}{\bar{\sigma}^2 M^2} \left[ (12u-1)\tilde{\omega} - 6\omega + \frac{8um_s^2(\omega + \tilde{\omega})}{\bar{\sigma} M^2} - \frac{4\omega\tilde{\omega}}{\bar{\sigma} m_B} \left( 1 - \frac{\tilde{m}_B^2}{2M^2} \right) \right. \\
& \left. \left. + \frac{8\omega}{\bar{\sigma}} \left( 1 - \frac{s}{2M^2} \right) \right] + \frac{4\tilde{Y}_A(\omega, \xi)}{\bar{\sigma}^3 M^4} [\omega\tilde{\omega}(2m_s - \tilde{\omega}) + 2u(\omega + \tilde{\omega})\tilde{\omega}^2] \right\} e^{-\frac{s}{M^2}} \Bigg\}, \tag{18}
\end{aligned}$$

$$\begin{aligned}
\tilde{A}_3(q^2) = & \frac{2f_B m_B(m_B + m_{K_2^*})}{f_{K_2^*} m_{K_2^*}^2} e^{\frac{m_{K_2^*}^2}{M^2}} \left\{ \int_0^{\sigma_0} d\sigma \omega' \left\{ \frac{\phi_+(\omega')}{\bar{\sigma}} + \frac{2\omega' - m_s}{\bar{\sigma}^2 M^2} [\tilde{\phi}_+(\omega') - \tilde{\phi}_-(\omega')] \right\} e^{-\frac{s}{M^2}} \right. \\
& + \int_0^{\sigma_0} \tilde{d\sigma} \left\{ \frac{[\Psi_A(\omega, \xi) - \Psi_V(\omega, \xi)][2u(m_s - 2\tilde{\omega} - \omega) - \omega]}{\bar{\sigma}^2 M^2} - \frac{2\Psi_V(\omega, \xi)[(1-u)\omega + u\tilde{\omega}]}{\bar{\sigma}^2 M^2} \right. \\
& + \frac{2uX_A(\omega, \xi)(\omega + \tilde{\omega})}{\bar{\sigma}^2 M^2} + \frac{\tilde{X}_A(\omega, \xi)}{\bar{\sigma}^2 M^2} \left[ \frac{1}{2} + 4u + \frac{2\omega(m_s + \tilde{\omega}) + 4um_s^2}{\bar{\sigma} M^2} + \frac{2\omega}{\bar{\sigma} m_B} \left( 1 - \frac{\tilde{m}_B^2}{2M^2} \right) \right] \\
& \left. \left. + \frac{2\tilde{Y}_A(\omega, \xi)}{\bar{\sigma}^3 M^4} [\omega(m_s - 2\tilde{\omega}) + 2u\tilde{\omega}(2\tilde{\omega} - m_s + 2\omega)] \right\} e^{-\frac{s}{M^2}} \right\}, \tag{19}
\end{aligned}$$

$$\begin{aligned}
T_1(q^2) = & -\frac{f_B m_B}{f_{K_2^*} m_{K_2^*}^2} e^{\frac{m_{K_2^*}^2}{M^2}} \left\{ \int_0^{\sigma_0} d\sigma \omega' (\omega' - m_B - m_s) \left\{ \frac{\phi_+(\omega')}{\bar{\sigma}} - \frac{m_s}{\bar{\sigma}^2 M^2} [\tilde{\phi}_+(\omega') - \tilde{\phi}_-(\omega')] \right\} \right. \\
& e^{-\frac{s}{M^2}} + \int_0^{\sigma_0} d\sigma \left\{ \left[ \frac{u}{\bar{\sigma}} + \frac{[\omega(m_B - \tilde{\omega} - m_s) - 2um_s(m_s + m_B - \tilde{\omega}) + 2u\omega(\tilde{\omega} - m_B)]}{\bar{\sigma}^2 M^2} \right] \right. \\
& [\Psi_A(\omega, \xi) - \Psi_V(\omega, \xi)] + \frac{2[(1-u)\omega(m_B - \tilde{\omega}) + u\tilde{\omega}(m_B + m_s - \tilde{\omega})] \Psi_V(\omega, \xi)}{\bar{\sigma}^2 M^2} \\
& + \frac{2u[\tilde{\omega}(\tilde{\omega} - m_B - m_s) + \omega(\tilde{\omega} - m_B)] X_A(\omega, \xi)}{\bar{\sigma}^2 M^2} + \frac{\tilde{X}_A(\omega, \xi)}{\bar{\sigma}^2 M^2} \left[ -\frac{6\omega + \tilde{\omega} + m_B + m_s}{2} \right. \\
& + 4u(\tilde{\omega} - m_B) + 2u\tilde{\omega} + \frac{2[m_B\omega(m_s - \tilde{\omega}) + 2um_s^2(\tilde{\omega} - m_B - m_s + \omega)]}{\bar{\sigma} M^2} \\
& + \frac{4\omega}{\bar{\sigma}} \left( 1 - \frac{s}{2M^2} \right) - \frac{2\omega(\tilde{\omega} + m_B - m_s)}{m_B \bar{\sigma}} \left( 1 - \frac{\tilde{m}_B^2}{2M^2} \right) \left. \right] + \frac{2\tilde{Y}_A(\omega, \xi)}{\bar{\sigma}^2 M^2} [(1-2u)\omega \\
& + (1-4u)\tilde{\omega} + \frac{m_s[\omega(m_s + \tilde{\omega} - m_B) + 2u\tilde{\omega}(m_s - \tilde{\omega} + m_B) - 2um_s\omega]}{\bar{\sigma} M^2} \left. \right] \left. \right\} e^{-\frac{s}{M^2}} \Bigg\} \quad (20)
\end{aligned}$$

$$\begin{aligned}
T_2(q^2) = & \frac{2f_B m_B^2}{f_{K_2^*} m_{K_2^*}^2 (m_B^2 - m_{K_2^*}^2)} e^{\frac{m_{K_2^*}^2}{M^2}} \left\{ \int_0^{\sigma_0} d\sigma \omega' \left\{ \frac{\phi_+(\omega')}{\bar{\sigma}} \left[ m_B(m_s - \omega') - s + \frac{\tilde{m}_B^2(\omega' + m_B - m_s)}{2m_B} \right] \right. \right. \\
& + \frac{m_s}{\bar{\sigma}^2} \left[ \frac{m_B(\omega' - m_s)}{M^2} + \frac{m_B + \omega' - m_s}{2m_B} \left( 1 - \frac{\tilde{m}_B^2}{M^2} \right) - \left( 1 - \frac{s}{M^2} \right) \right] \left[ \tilde{\phi}_+(\omega') - \tilde{\phi}_-(\omega') \right] \Big\} \\
& e^{-\frac{s}{M^2}} + \int_0^{\sigma_0} \tilde{d\sigma} \left\{ [\Psi_A(\omega, \xi) - \Psi_V(\omega, \xi)] \left[ -\frac{u}{\bar{\sigma}} \left( m_B - \frac{\tilde{m}_B^2}{2m_B} \right) \right. \right. \\
& + \frac{m_B [2um_s(m_s - \tilde{\omega}) + \omega(m_s + \tilde{\omega}) - 2u\omega\tilde{\omega}]}{\bar{\sigma}^2 M^2} + \frac{2u(m_s + \omega) - \omega}{\bar{\sigma}^2} \left( 1 - \frac{s}{M^2} \right) \\
& + \frac{\omega(\tilde{\omega} + m_B + m_s) - 2um_s(\tilde{\omega} + m_B - m_s) - 2u\omega(m_B + \tilde{\omega})}{2m_B \bar{\sigma}^2} \left( 1 - \frac{\tilde{m}_B^2}{M^2} \right) \Big] \\
& + \frac{2\Psi_V(\omega, \xi)}{\bar{\sigma}^2} \left[ \frac{m_B [(1-u)\omega\tilde{\omega} + u\tilde{\omega}(\tilde{\omega} - m_s)]}{M^2} - [(1-u)\omega + u\tilde{\omega}] \left( 1 - \frac{s}{M^2} \right) \right. \\
& + \frac{(1-u)\omega(m_B + \tilde{\omega}) + u\tilde{\omega}(\tilde{\omega} + m_B - m_s)}{2m_B} \left( 1 - \frac{\tilde{m}_B^2}{M^2} \right) \Big] + \frac{2uX_A(\omega, \xi)}{\bar{\sigma}^2} \\
& \left[ \frac{m_B \tilde{\omega}(m_s - \tilde{\omega} - \omega)}{M^2} - \frac{\tilde{\omega}(\tilde{\omega} + m_B - m_s) + \omega(\tilde{\omega} + m_B)}{2m_B} \left( 1 - \frac{\tilde{m}_B^2}{M^2} \right) + (\tilde{\omega} + \omega) \right. \\
& \left. \left( 1 - \frac{s}{M^2} \right) \right] + \frac{\tilde{X}_A(\omega, \xi)}{\bar{\sigma}^2} \left[ \frac{m_B(m_s + 6\omega + \tilde{\omega}) - 12um_B\tilde{\omega}}{2M^2} + \frac{1 + 8u}{2} \left( 1 - \frac{s}{M^2} \right) \right. \\
& + \frac{4um_s^2 m_B(m_s - \tilde{\omega} - \omega)}{\bar{\sigma} M^4} + \frac{6\omega + \tilde{\omega} + m_s - m_B - 8u(\tilde{\omega} + m_B) - 4u\tilde{\omega}}{4m_B} \left( 1 - \frac{\tilde{m}_B^2}{M^2} \right) \\
& + \frac{8um_s^2 - 4\omega(m_B + m_s - \tilde{\omega})}{\bar{\sigma} M^2} \left( 1 - \frac{s}{2M^2} \right) - \frac{4um_s^2(\tilde{\omega} + m_B - m_s + \omega)}{m_B \bar{\sigma} M^2} \left( 1 - \frac{\tilde{m}_B^2}{2M^2} \right) \\
& - \frac{2\omega(m_B + m_s - \tilde{\omega})}{m_B^2 \bar{\sigma}} \left( \frac{1}{2} - \frac{\tilde{m}_B^2}{M^2} + \frac{\tilde{m}_B^4}{4M^4} \right) \Big] + \frac{2\tilde{Y}_A(\omega, \xi)}{\bar{\sigma}^2} \left[ \frac{m_B [2u\tilde{\omega} + (2u-1)(\omega + \tilde{\omega})]}{M^2} \right. \\
& + \frac{(2u-1)(\omega + \tilde{\omega}) + 2u\tilde{\omega}}{2m_B} \left( 1 - \frac{\tilde{m}_B^2}{M^2} \right) + \frac{m_s m_B [2u\tilde{\omega}(\tilde{\omega} - m_s) - \omega(m_s + \tilde{\omega}) + 2um_s\omega]}{\bar{\sigma} M^4} \\
& + \frac{m_s [2u\tilde{\omega}(m_B + \tilde{\omega} - m_s) - \omega(\tilde{\omega} + m_B + m_s) + 2um_s\omega]}{m_B \bar{\sigma} M^2} \left( 1 - \frac{\tilde{m}_B^2}{2M^2} \right) \\
& \left. \left. + \frac{2m_s(\omega - 2u\tilde{\omega})}{\bar{\sigma} M^2} \left( 1 - \frac{s}{2M^2} \right) \right] \right\} e^{-\frac{s}{M^2}} \Big\} , \tag{21}
\end{aligned}$$



$$\begin{aligned}
\tilde{T}_3(q^2) = & -\frac{2f_B}{f_{K_2^*}m_{K_2^*}^2}e^{\frac{m_{K_2^*}^2}{M^2}}\left\{\int_0^{\sigma_0}d\sigma\omega'\left\{\frac{\phi_+(\omega')m_B(\omega'-m_s)}{\bar{\sigma}}+\frac{1}{\bar{\sigma}}\left[\tilde{\phi}_+(\omega')-\tilde{\phi}_-(\omega')\right]\left[m_B\right.\right.\right. \\
& +\left.\left.\frac{m_B\omega'(\omega'-m_s)}{\bar{\sigma}M^2}+\frac{2\omega'-m_B}{\bar{\sigma}}\left(1-\frac{s}{M^2}\right)\right]\right\}e^{-\frac{s}{M^2}}+\int_0^{\sigma_0}\tilde{d\sigma}\left\{[\Psi_A(\omega,\xi)-\Psi_V(\omega,\xi)]\right. \\
& \left[-\frac{um_B}{\bar{\sigma}}+\frac{m_B[2u\tilde{\omega}(m_s-\tilde{\omega})-\omega(m_s+\tilde{\omega})+2u\omega\tilde{\omega}]}{\bar{\sigma}^2M^2}-\frac{2u[(2\tilde{\omega}-m_B)+2\omega]}{\bar{\sigma}^2}\right. \\
& \left.\left.\left(1-\frac{s}{M^2}\right)+\frac{2u\omega}{\bar{\sigma}^2}\left(1-\frac{\tilde{m}_B^2}{M^2}\right)\right]+\frac{2m_B\tilde{\omega}[u(m_s-\tilde{\omega})+(u-1)\omega]\Psi_V(\omega,\xi)}{\bar{\sigma}^2M^2}\right. \\
& +\frac{2u[m_B\tilde{\omega}(\tilde{\omega}-m_s+\omega)]X_A(\omega,\xi)}{\bar{\sigma}^2M^2}+\frac{\tilde{X}_A(\omega,\xi)}{\bar{\sigma}^2M^2}\left[-\frac{m_B(6\omega+\tilde{\omega}+m_s-12u\tilde{\omega})}{2}\right. \\
& +\frac{4um_s^2m_B(\tilde{\omega}-m_s+\omega)}{\bar{\sigma}M^2}+\frac{4\omega(m_B+2m_s)}{\bar{\sigma}}\left(1-\frac{s}{2M^2}\right)-\frac{2m_B\omega(m_s+\tilde{\omega})}{\bar{\sigma}m_B}\left(1-\frac{\tilde{m}_B^2}{2M^2}\right)\left.\right] \\
& +\frac{2\tilde{Y}_A(\omega,\xi)}{\bar{\sigma}^2M^2}\left[m_B[(1-2u)(\omega+\tilde{\omega})-2u\tilde{\omega}]+\frac{m_Bm_s[\omega(m_s+\tilde{\omega})+2u\tilde{\omega}(m_s-\tilde{\omega})-2um_s\omega]}{\bar{\sigma}M^2}\right. \\
& \left.\left.-\frac{2\tilde{\omega}[2u\tilde{\omega}+(2u-1)\omega]}{\bar{\sigma}}\left(1-\frac{\tilde{m}_B^2}{2M^2}\right)+\frac{4\tilde{\omega}[2u\tilde{\omega}+(2u-1)\omega]}{\bar{\sigma}}\left(1-\frac{s}{2M^2}\right)\right]\right\}e^{-\frac{s}{M^2}}\Bigg\}, \tag{22}
\end{aligned}$$

where

$$\begin{aligned}
\int_0^{\sigma_0}\tilde{d\sigma} &= \int_0^{\sigma_0}d\sigma\int_0^{\sigma m_B}d\omega\int_{\sigma m_B-\omega}^{\infty}\frac{d\xi}{\xi}, \\
s &= m_B^2\sigma-\frac{\sigma q^2-m_s^2}{\bar{\sigma}}, \quad \omega'=\sigma m_B, \quad \bar{\sigma}=1-\sigma, \quad \tilde{\omega}=\omega+u\xi, \\
\sigma_0 &= \frac{s_0^{K_2^*}+m_B^2-q^2-\sqrt{(s_0^{K_2^*}+m_B^2-q^2)^2-4(s_0^{K_2^*}-m_s^2)m_B^2}}{2m_B^2}, \\
u &= \frac{\sigma m_B-\omega}{\xi}, \quad \tilde{m}_B^2=m_B^2(1+\sigma)-\frac{q^2-m_s^2}{\bar{\sigma}}, \\
\tilde{X}_A(\omega,\xi) &= \int_0^\omega d\lambda X_A(\lambda,\xi), \quad \tilde{Y}_A(\omega,\xi)=\int_0^\omega d\lambda Y_A(\lambda,\xi), \\
\tilde{\phi}_\pm(\omega) &= \int_0^\omega d\lambda \phi_\pm(\lambda). \tag{23}
\end{aligned}$$

With a simple replacement,

$$\begin{aligned}
m_s &\rightarrow 0, \quad m_{K_2^*}\rightarrow m_{a_2}, \quad f_{K_2^*}\rightarrow f_{a_2}, \quad s_0^{K_2^*}\rightarrow s_0^{a_2}, \\
m_s &\rightarrow 0, \quad m_{K_2^*}\rightarrow m_{f_2}, \quad f_{K_2^*}\rightarrow f_{f_2}, \quad s_0^{K_2^*}\rightarrow s_0^{f_2}, \tag{24}
\end{aligned}$$

we can obtain the corresponding sum rules for the  $B \rightarrow a_2(1320)$  and  $B \rightarrow f_2(1270)$  form-factors, respectively.

In Ref.[21], Lange and Neubert observe that the evolution effects drive the light-cone distribution amplitude  $\phi_+(\omega)$  toward a linear growth at the origin and generate a radiative tail that falls off slower than  $\frac{1}{\omega}$ , even if the initial function has an arbitrarily rapid falloff. The normalization integral of the  $\phi_+(\omega)$  is ultraviolet divergent. In this article, we derive the sum rules without the radiative  $\mathcal{O}(\alpha_s)$  corrections, the ultraviolet behavior of the  $\phi_+(\omega)$  plays no role at the leading order  $\mathcal{O}(1)$ . Furthermore, the duality thresholds in the sum rules are well below the region where the

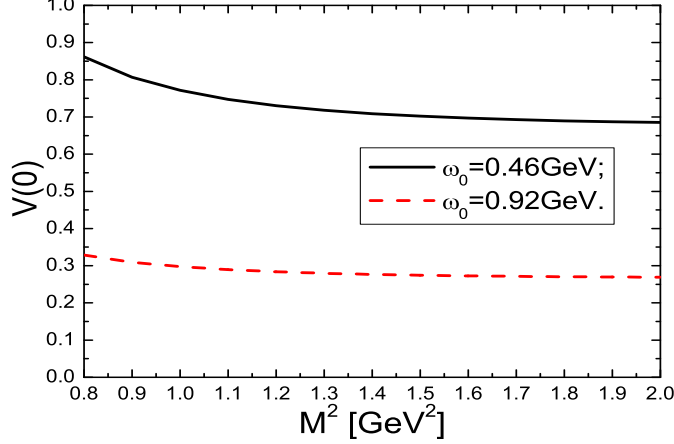


Figure 2: The form-factor  $V_{BK_2^*}(0)$  with variation of the Borel parameter  $M^2$ , other parameters are taken to be the central values.

effect of the tail becomes noticeable. The nontrivial renormalization of the  $B$ -meson light-cone distribution amplitudes is so far known only for the  $\phi_+(\omega)$ , we use the light-cone distribution amplitudes of order  $\mathcal{O}(1)$ , which satisfy all QCD constraints.

### 3 Numerical result and discussion

The input parameters for the  $B$ -meson light-cone distribution amplitudes are taken as  $\omega_0 = \lambda_B(\mu) = (0.46 \pm 0.11) \text{ GeV}$ ,  $\mu = 1 \text{ GeV}$  [22],  $\lambda_E^2 = (0.11 \pm 0.06) \text{ GeV}^2$  [19],  $m_B = 5.279 \text{ GeV}$ , and  $f_B = (0.18 \pm 0.02) \text{ GeV}$  [33].

The masses, decay constants (or pole residues), threshold parameters and Borel parameters of the tensor mesons  $f_2(1270)$ ,  $a_2(1320)$  and  $K_2^*(1430)$  are determined by the conventional two-point QCD sum rules. The values are  $m_{K_2^*} = (1.43 \pm 0.01) \text{ GeV}$ ,  $m_{a_2} = (1.31 \pm 0.01) \text{ GeV}$ ,  $m_{f_2} = (1.27 \pm 0.01) \text{ GeV}$ ,  $f_{K_2^*} = (0.118 \pm 0.005) \text{ GeV}$ ,  $f_{a_2} = (0.107 \pm 0.006) \text{ GeV}$ ,  $f_{f_2} = (0.102 \pm 0.006) \text{ GeV}$ ,  $s_0^{K_2^*} = 3.13 \text{ GeV}^2$ ,  $s_0^{a_2} = 2.70 \text{ GeV}^2$ ,  $s_0^{f_2} = 2.53 \text{ GeV}^2$ ,  $M_{K_2^*}^2 = (1.2 - 1.6) \text{ GeV}^2$ ,  $M_{a_2}^2 = (1.0 - 1.4) \text{ GeV}^2$  and  $M_{f_2}^2 = (1.0 - 1.4) \text{ GeV}^2$  [34]. The tensor meson  $K_2^*(1430)$  was originally studied with the QCD sum rules by T. M. Aliev et al [35]. In the conventional two-point QCD sum rules for the tensor mesons, we determine the Borel windows by imposing the two criteria (pole dominance and convergence of the operator product expansion) to reproduce the experimental values of the masses [34]. In this article, we take the tensor currents  $J_{\mu\nu}(x)$  to interpolate the tensor mesons  $K_2^*(1430)$ ,  $a_2(1320)$  and  $f_2(1270)$  as in the conventional two-point QCD sum rules, and expect that the parameters survive.

Taking into account all uncertainties of the relevant parameters, we obtain the numerical values of the form-factors  $V(q^2)$ ,  $A_1(q^2)$ ,  $\tilde{A}_2(q^2)$ ,  $\tilde{A}_3(q^2)$ ,  $T_1(q^2)$ ,  $T_2(q^2)$  and  $\tilde{T}_3(q^2)$ . The values of the form-factors  $V(0)$ ,  $A_1(0)$ ,  $\tilde{A}_2(0)$ ,  $\tilde{A}_3(0)$ ,  $T_1(0)$ ,  $T_2(0)$  are very stable with variations of the Borel parameters in a large range, the uncertainties originate from the Borel parameters are small (for example, see Fig.2); while the values of the form-factors  $\tilde{T}_3(0)$  are not stable enough with the variation of the Borel parameter even in the Borel window. The form-factors can be parameterized

in the double-pole form,

$$F_i(q^2) = \frac{F_i(0)}{1 + a_F q^2/m_B^2 + b_F q^4/m_B^4}, \quad (25)$$

where the  $F_i(q^2)$  denote the  $V(q^2)$ ,  $A_1(q^2)$ ,  $\tilde{A}_2(q^2)$ ,  $\tilde{A}_3(q^2)$ ,  $T_1(q^2)$ ,  $T_2(q^2)$  and  $\tilde{T}_3(q^2)$ , the  $a_F$  and  $b_F$  are the corresponding coefficients. The values of the form-factors at zero momentum transfer and the  $a_F$ ,  $b_F$  are presented in Table 1. In Table 2, we present the central values of the ratios among the form-factors at zero momentum transfer.

In this article, we calculate the uncertainties  $\delta$  with the formula

$$\delta = \sqrt{\sum_i \left( \frac{\partial f}{\partial x_i} \right)^2 \Big|_{x_i=\bar{x}_i} (x_i - \bar{x}_i)^2}, \quad (26)$$

where the  $f$  denotes the  $B \rightarrow T$  form-factors, the  $x_i$  denotes the input parameters  $\omega_0$ ,  $\lambda_E^2$ ,  $f_{K_2^*}$ ,  $\dots$ . As the partial derivatives  $\frac{\partial f}{\partial x_i}$  are difficult to carry out analytically, we take the approximation  $\left( \frac{\partial f}{\partial x_i} \right)^2 (x_i - \bar{x}_i)^2 \approx [f(\bar{x}_i \pm \Delta x_i) - f(\bar{x}_i)]^2$  in the numerical calculations, and take into account the values  $[f(\bar{x}_i + \Delta x_i) - f(\bar{x}_i)]^2 \neq [f(\bar{x}_i - \Delta x_i) - f(\bar{x}_i)]^2$ .

In calculation, we observe that the dominating contributions in the sum rules for the form-factors come from the two-particle  $B$ -meson light-cone distribution amplitudes, the contributions from the three-particle  $B$ -meson light-cone distribution amplitudes are of minor importance<sup>2</sup>. This is the prominent advantage of the  $B$ -meson light-cone QCD sum rules. It is not un-expected that the dominating uncertainty comes from the parameter  $\omega_0$  (or  $\lambda_B$ ), which determines the line-shapes of the two-particle and three-particle light-cone distribution amplitudes. We can take the value  $\omega_0 = \lambda_B = (0.46 \pm 0.11)$  GeV from the QCD sum rules in Ref.[22], where the  $B$ -meson light-cone distribution amplitude  $\phi_+(\omega)$  is parameterized by the matrix element of the bilocal operator at imaginary light-cone separation.

In Tables 3-5, we also present the values (and the ratios among the central values) of the  $B \rightarrow T$  form-factors from other theoretical calculations, such as the improved version of the ISGW model [5], the covariant light-front quark model [6], the light-cone sum rules approach [7], the large energy effective theory [8, 9, 10] and the perturbative QCD approach [11]. From those tables, we can see that the central values of the present predictions are at least (or almost) twice as large as the existing estimations. If we take the  $\phi_+(\omega)$  to be the more elaborated model (the BIK model) suggested in Ref.[22], the values of the  $B \rightarrow T$  form-factors at  $q^2 = 0$  are even larger, see Table 1, where only the central values are presented for simplicity. The two-particle  $B$ -meson distribution amplitudes in the  $k_T$  factorization formalism have the same Gaussian-like distribution amplitudes  $\phi_+(\omega)$  and  $\phi_-(\omega)$  besides additional factors describing the  $b$ -parameter dependence [32]. In this article, we do not factorize out the  $k_T$  explicitly, and cannot take the distribution amplitudes in  $k_T$  factorization formalism.

The form-factors  $T_1(q^2)$ ,  $T_2(q^2)$  and  $T_3(q^2)$  are related to the radiative decays  $B \rightarrow T\gamma$ . The branching ratio of the radiative decay  $B \rightarrow K_2^*(1430)\gamma$  can be written as

$$\mathcal{B}(B \rightarrow K_2^* \gamma) = \tau_B \frac{G_F^2 \alpha_{\text{em}} m_B^5 m_b^2 |V_{tb} V_{ts}^*|^2}{256 \pi^4 m_{K_2^*}^2} \left( 1 - \frac{m_{K_2^*}^2}{m_B^2} \right)^5 |C_7 + A^{(1)}|^2 |T_1^{BK_2^*}(0)|^2, \quad (27)$$

where the  $V_{tb}$  and  $V_{ts}$  are the CKM matrix elements, the  $C_7$  is the Wilson coefficient for the operator  $O_{7\gamma}$ , the  $A^{(1)}$  is the perturbative  $\mathcal{O}(\alpha_s)$  corrections [36], the  $\tau_B$  is the  $B$ -meson lifetime. In Eq.(27),

<sup>2</sup>The form-factor  $\tilde{T}_3(q^2)$  is defined by  $\tilde{T}_3(q^2) = T_2(q^2) - T_3(q^2) \left( 1 - q^2 / (m_B^2 - m_{K_2^*}^2) \right)$ . In the sum rules for the  $T_2(q^2)$  and  $\tilde{T}_3(q^2)$ , the dominating contributions come from the two-particle and three-particle  $B$ -meson light-cone distribution amplitudes, respectively. It is obvious that the form-factor  $T_3(q^2)$  acquires its value mainly from the two-particle  $B$ -meson light-cone distribution amplitudes.

$\omega_0$		$F_{BK_s^*}(0)$	$[a_F, b_F]$	$F_{Ba_2}(0)$	$[a_F, b_F]$	$F_{Bf_2}(0)$	$[a_F, b_F]$
$V$	$\tilde{\lambda}_B$	$0.71^{+0.29}_{-0.20}$	$[-2.52, 1.75]$	$0.60^{+0.28}_{-0.17}$	$[-2.58, 1.86]$	$0.57^{+0.26}_{-0.16}$	$[-2.59, 1.90]$
	$2\tilde{\lambda}_B$	$0.28^{+0.15}_{-0.09}$	$[-2.93, 2.33]$	$0.21^{+0.13}_{-0.07}$	$[-2.93, 2.38]$	$0.20^{+0.11}_{-0.07}$	$[-2.93, 2.40]$
	BIK	0.76		0.63		0.59	
$A_1$	$\tilde{\lambda}_B$	$0.43^{+0.19}_{-0.12}$	$[-1.38, 0.49]$	$0.37^{+0.16}_{-0.11}$	$[-1.40, 0.53]$	$0.35^{+0.17}_{-0.10}$	$[-1.43, 0.54]$
	$2\tilde{\lambda}_B$	$0.17^{+0.09}_{-0.05}$	$[-1.78, 0.65]$	$0.13^{+0.07}_{-0.04}$	$[-1.75, 0.67]$	$0.12^{+0.07}_{-0.04}$	$[-1.76, 0.69]$
	BIK	0.46		0.38		0.36	
$-\tilde{A}_2$	$\tilde{\lambda}_B$	$0.18^{+0.06}_{-0.05}$	$[-3.47, 3.64]$	$0.14^{+0.06}_{-0.04}$	$[-3.54, 3.81]$	$0.13^{+0.05}_{-0.04}$	$[-3.57, 3.88]$
	$2\tilde{\lambda}_B$	$0.07^{+0.04}_{-0.02}$	$[-3.95, 4.62]$	$0.05^{+0.03}_{-0.01}$	$[-3.95, 4.68]$	$0.05^{+0.03}_{-0.02}$	$[-3.95, 4.69]$
	BIK	0.20		0.16		0.14	
$\tilde{A}_3$	$\tilde{\lambda}_B$	$1.07^{+0.52}_{-0.36}$	$[-2.16, 1.52]$	$0.89^{+0.51}_{-0.35}$	$[-2.18, 1.59]$	$0.86^{+0.50}_{-0.34}$	$[-2.23, 1.61]$
	$2\tilde{\lambda}_B$	$0.28^{+0.26}_{-0.15}$	$[-1.88, 2.93]$	$0.20^{+0.23}_{-0.14}$	$[-1.71, 4.03]$	$0.19^{+0.22}_{-0.12}$	$[-1.81, 3.11]$
	BIK	1.17		0.94		0.90	
$T_1$	$\tilde{\lambda}_B$	$0.54^{+0.22}_{-0.15}$	$[-2.45, 1.67]$	$0.46^{+0.21}_{-0.14}$	$[-2.51, 1.78]$	$0.44^{+0.20}_{-0.13}$	$[-2.54, 1.82]$
	$2\tilde{\lambda}_B$	$0.21^{+0.11}_{-0.07}$	$[-2.86, 2.22]$	$0.16^{+0.09}_{-0.05}$	$[-2.86, 2.28]$	$0.15^{+0.09}_{-0.05}$	$[-2.87, 2.30]$
	BIK	0.57		0.47		0.45	
$T_2$	$\tilde{\lambda}_B$	$0.54^{+0.23}_{-0.15}$	$[-1.32, 0.49]$	$0.46^{+0.21}_{-0.14}$	$[-1.34, 0.52]$	$0.44^{+0.20}_{-0.13}$	$[-1.37, 0.53]$
	$2\tilde{\lambda}_B$	$0.21^{+0.12}_{-0.07}$	$[-1.71, 0.61]$	$0.16^{+0.10}_{-0.05}$	$[-1.68, 0.64]$	$0.15^{+0.09}_{-0.05}$	$[-1.70, 0.65]$
	BIK	0.58		0.47		0.45	
$\tilde{T}_3$	$\tilde{\lambda}_B$	$0.09^{+0.04}_{-0.03}$	$[-1.86, 1.11]$	$0.07^{+0.03}_{-0.04}$	$[-1.93, 1.14]$	$0.06^{+0.03}_{-0.03}$	$[-1.95, 1.20]$
	$2\tilde{\lambda}_B$	$0.09^{+0.03}_{-0.03}$	$[-2.74, 1.86]$	$0.07^{+0.03}_{-0.02}$	$[-2.80, 2.02]$	$0.06^{+0.03}_{-0.01}$	$[-2.80, 2.05]$
	BIK	0.08		0.06		0.06	
$A_2$	$\tilde{\lambda}_B$	$0.45^{+0.26}_{-0.18}$		$0.38^{+0.26}_{-0.18}$		$0.37^{+0.25}_{-0.17}$	
	$2\tilde{\lambda}_B$	$0.11^{+0.13}_{-0.08}$		$0.08^{+0.11}_{-0.07}$		$0.07^{+0.11}_{-0.06}$	
$A_3$	$\tilde{\lambda}_B$	$0.40^{+0.57}_{-0.37}$		$0.35^{+0.56}_{-0.39}$		$0.32^{+0.59}_{-0.37}$	
	$2\tilde{\lambda}_B$	$0.25^{+0.27}_{-0.16}$		$0.21^{+0.24}_{-0.15}$		$0.20^{+0.25}_{-0.14}$	
$T_3$	$\tilde{\lambda}_B$	$0.45^{+0.23}_{-0.15}$		$0.39^{+0.21}_{-0.14}$		$0.38^{+0.20}_{-0.13}$	
	$2\tilde{\lambda}_B$	$0.12^{+0.12}_{-0.08}$		$0.09^{+0.10}_{-0.05}$		$0.09^{+0.09}_{-0.05}$	

Table 1: The values of the form-factors at zero momentum transfer and the parameters of the  $B \rightarrow T$  form-factors, where the BIK denotes the  $\phi_+(\omega)$  is taken as the BIK light-cone distribution amplitude.

$\omega_0$	$\widehat{F_{BK_2^*}}(0)$	$\widehat{F_{Ba_2}}(0)$	$\widehat{F_{Bf_2}}(0)$
$\widehat{V}$	$\lambda_B$	1.00	1.00
	$2\tilde{\lambda}_B$	1.00	1.00
	BIK	1.00	1.00
$\widehat{A_1}$	$\tilde{\lambda}_B$	0.61	0.62
	$2\tilde{\lambda}_B$	0.61	0.62
	BIK	0.61	0.60
$-\widehat{\tilde{A}_2}$	$\tilde{\lambda}_B$	0.25	0.23
	$2\tilde{\lambda}_B$	0.25	0.24
	BIK	0.26	0.24
$\widehat{\tilde{A}_3}$	$\tilde{\lambda}_B$	1.51	1.48
	$2\tilde{\lambda}_B$	1.00	0.95
	BIK	1.54	1.49
$\widehat{T_1}$	$\tilde{\lambda}_B$	0.76	0.77
	$2\tilde{\lambda}_B$	0.75	0.76
	BIK	0.75	0.75
$\widehat{T_2}$	$\tilde{\lambda}_B$	0.76	0.77
	$2\tilde{\lambda}_B$	0.75	0.76
	BIK	0.76	0.75
$\widehat{\tilde{T}_3}$	$\tilde{\lambda}_B$	0.13	0.12
	$2\tilde{\lambda}_B$	0.32	0.33
	BIK	0.11	0.10
$\widehat{A_2}$	$\tilde{\lambda}_B$	0.63	0.63
	$2\tilde{\lambda}_B$	0.39	0.38
$\widehat{A_3}$	$\lambda_B$	0.56	0.58
	$2\tilde{\lambda}_B$	0.89	1.00
$\widehat{T_3}$	$\tilde{\lambda}_B$	0.63	0.65
	$2\tilde{\lambda}_B$	0.43	0.43

Table 2: The central values of the ratios among the form-factors at zero momentum transfer and the parameters of the form-factors, where the BIK denotes the  $\phi_+(\omega)$  is taken as the BIK light-cone distribution amplitude,  $\widehat{F_{BK_2^*}}(0) = \frac{F_{BK_2^*}(0)}{V_{BK_2^*}(0)}$ ,  $\widehat{F_{Ba_2}}(0) = \frac{F_{Ba_2}(0)}{V_{Ba_2}(0)}$  and  $\widehat{F_{Bf_2}}(0) = \frac{F_{Bf_2}(0)}{V_{Bf_2}(0)}$ .

	Ref.[5]	Ref.[6]	Ref.[7]	Refs.[8, 9, 10]	Ref.[11]	This Work	
$V(0)$	0.38	0.29	$0.16 \pm 0.04$	$0.21 \pm 0.03$	$0.21^{+0.06}_{-0.05}$	$0.71^{+0.29}_{-0.20}$	$0.28^{+0.15}_{-0.09}$
$A_1(0)$	0.24	0.22	$0.14 \pm 0.04$	$0.14 \pm 0.02$	$0.13^{+0.04}_{-0.03}$	$0.43^{+0.19}_{-0.12}$	$0.17^{+0.09}_{-0.05}$
$A_2(0)$	0.22	0.21	$0.05 \pm 0.04$	$0.14 \pm 0.02$	$0.08^{+0.03}_{-0.02}$	$0.45^{+0.26}_{-0.18}$	$0.11^{+0.13}_{-0.08}$
$A_0(0)$	0.27	0.23	$0.25 \pm 0.06$	$0.15 \pm 0.02$	$0.18^{+0.05}_{-0.04}$	$0.40^{+0.57}_{-0.37}$	$0.25^{+0.27}_{-0.16}$
$\widehat{A}_1(0)$	0.63	0.76	0.88	0.67	0.62	0.61 [0.61]	
$\widehat{A}_2(0)$	0.58	0.72	0.31	0.67	0.38	0.63 [0.39]	
$\widehat{A}_0(0)$	0.71	0.79	1.56	0.71	0.86	0.56 [0.89]	

Table 3: The values of the  $B \rightarrow K_2^*(1430)$  form-factors at zero momentum transfer from different theoretical approaches, the values in the bracket correspond to  $\omega_0 = 2\tilde{\lambda}_B$ . We take the central values of the ratios  $\widehat{A}_1(0) = \frac{A_1(0)}{V(0)}$ ,  $\widehat{A}_2(0) = \frac{A_2(0)}{V(0)}$  and  $\widehat{A}_0(0) = \frac{A_0(0)}{V(0)}$ .

	Ref.[5]	Ref.[6]	Ref.[7]	Refs.[8, 9, 10]	Ref.[11]	This Work	
$V(0)$	0.32	0.28	$0.18 \pm 0.02$	$0.18 \pm 0.03$	$0.18^{+0.05}_{-0.04}$	$0.60^{+0.28}_{-0.17}$	$0.21^{+0.13}_{-0.07}$
$A_1(0)$	0.16	0.21	$0.14 \pm 0.02$	$0.13 \pm 0.02$	$0.11^{+0.03}_{-0.03}$	$0.37^{+0.16}_{-0.11}$	$0.13^{+0.07}_{-0.04}$
$A_2(0)$	0.14	0.19	$0.09 \pm 0.02$	$0.13 \pm 0.02$	$0.06^{+0.02}_{-0.01}$	$0.38^{+0.26}_{-0.18}$	$0.08^{+0.11}_{-0.07}$
$A_0(0)$	0.20	0.24	$0.21 \pm 0.04$	$0.14 \pm 0.02$	$0.18^{+0.06}_{-0.04}$	$0.35^{+0.56}_{-0.39}$	$0.21^{+0.24}_{-0.15}$
$\widehat{A}_1(0)$	0.50	0.75	0.78	0.72	0.61	0.62 [0.62]	
$\widehat{A}_2(0)$	0.44	0.68	0.50	0.72	0.33	0.63 [0.38]	
$\widehat{A}_0(0)$	0.63	0.86	1.17	0.78	1.00	0.58 [1.00]	

Table 4: The values of the  $B \rightarrow a_2(1320)$  form-factors at zero momentum transfer from different theoretical approaches, the values in the bracket correspond to  $\omega_0 = 2\tilde{\lambda}_B$ . We take the central values of the ratios  $\widehat{A}_1(0) = \frac{A_1(0)}{V(0)}$ ,  $\widehat{A}_2(0) = \frac{A_2(0)}{V(0)}$  and  $\widehat{A}_0(0) = \frac{A_0(0)}{V(0)}$ .

	Ref.[5]	Ref.[6]	Ref.[7]	Refs.[8, 9, 10]	Ref.[11]	This Work	
$V(0)$	0.32	0.28	$0.18 \pm 0.02$	$0.18 \pm 0.02$	$0.12^{+0.03}_{-0.03}$	$0.57^{+0.26}_{-0.16}$	$0.20^{+0.11}_{-0.07}$
$A_1(0)$	0.16	0.21	$0.14 \pm 0.02$	$0.12 \pm 0.02$	$0.08^{+0.02}_{-0.02}$	$0.35^{+0.17}_{-0.10}$	$0.12^{+0.07}_{-0.04}$
$A_2(0)$	0.14	0.19	$0.10 \pm 0.02$	$0.13 \pm 0.02$	$0.04^{+0.01}_{-0.01}$	$0.37^{+0.25}_{-0.17}$	$0.07^{+0.11}_{-0.06}$
$A_0(0)$	0.20	0.25	$0.20 \pm 0.04$	$0.13 \pm 0.02$	$0.13^{+0.04}_{-0.03}$	$0.32^{+0.59}_{-0.37}$	$0.20^{+0.25}_{-0.14}$
$\widehat{A}_1(0)$	0.50	0.75	0.78	0.67	0.67	0.61 [0.60]	
$\widehat{A}_2(0)$	0.44	0.68	0.56	0.72	0.33	0.65 [0.35]	
$\widehat{A}_0(0)$	0.63	0.89	1.11	0.72	1.08	0.56 [1.00]	

Table 5: The values of the  $B \rightarrow f_2(1270)$  form-factors at zero momentum transfer from different theoretical approaches, the values in the bracket correspond to  $\omega_0 = 2\tilde{\lambda}_B$ . We take the central values of the ratios  $\widehat{A}_1(0) = \frac{A_1(0)}{V(0)}$ ,  $\widehat{A}_2(0) = \frac{A_2(0)}{V(0)}$  and  $\widehat{A}_0(0) = \frac{A_0(0)}{V(0)}$ .

we take into account the factorizable contribution and its perturbative  $\mathcal{O}(\alpha_s)$  corrections in the standard model. From the experimental data of the radiative decays  $B^+ \rightarrow K_2^{*+}(1430)\gamma$  and  $B^0 \rightarrow K_2^{*0}(1430)\gamma$  [1, 2], we can obtain the value  $T_1^{BK_2^*}(0) = 0.16 \pm 0.02_{-0.01}^{+0.00}$ , which relates to the parameter  $\zeta_{\perp}^{BK_2^*}(0) = \frac{\bar{p}_{K_2^*}}{m_{K_2^*}} T_1^{BK_2^*}(0) = 0.27 \pm 0.03_{-0.01}^{+0.00}$  in Refs.[26, 27], the errors originate from the uncertainties of the experimental data and the  $b$ -quark pole mass, respectively. Compared with the experimental value, the present prediction  $T_1(0) = 0.54_{-0.15}^{+0.22}$  seems too large, see Table 1.

There are two possible explanations for the apparent discrepancy. One possibility is that the non-factorizable contributions we have neglected in extracting the form-factor  $T_1^{BK_2^*}(0)$  from the experimental data (for example, the contributions of the diagrams where the soft gluon is emitted from the intermediate  $c\bar{c}$  loops and then absorbed by the  $B$ -meson) are large enough. If we take into account them consistently, the value extracted from the experimental data maybe compatible with the present calculation. The non-factorizable contributions of the soft-gluon emitted from the intermediate  $c\bar{c}$  loops have been studied for the  $B \rightarrow K\gamma$  and  $B \rightarrow K^*\gamma$  decays [37], while the corresponding non-factorizable soft contributions of the  $B \rightarrow K_2^*(1430)\gamma$  decay have not been calculated yet. The other possibility is that the non-factorizable contributions in the decay  $B \rightarrow K_2^*(1430)\gamma$  are small enough to be neglected, the form-factor  $T_1^{BK_2^*}(0)$  extracted from the experimental data is precise enough, the apparent discrepancy is due to the shortcoming of the theory. In the following, we perform detailed discussions about the second possibility.

In calculation, we observe that if we double the value of the parameter  $\omega_0$ , i.e. taking  $\omega_0 = \lambda_B = 2\tilde{\lambda}_B = (0.92 \pm 0.22)$  GeV instead of  $\omega_0 = \lambda_B = \tilde{\lambda}_B = (0.46 \pm 0.11)$  GeV (here we introduce a new parameter  $\tilde{\lambda}_B$  to avoid confusion), we can obtain the value  $T_1(0) = 0.21_{-0.07}^{+0.11}$ , which is consistent with the theoretical estimations  $0.17_{-0.04}^{+0.05}$ ,  $0.19 \pm 0.04$  and  $0.28$  from the perturbative QCD [11], the light-cone QCD sum rules [38] and the covariant light-front quark model [39], respectively, or the value  $0.16 \pm 0.02_{-0.01}^{+0.00}$  extracted from the experimental data [26, 27]. The values (and the ratios among the central values) of the  $B \rightarrow T$  form-factors with  $\omega_0 = 2\tilde{\lambda}_B$  are presented in Tables 1-5, from the tables, we can see that the present predictions of the values of the form-factors are compatible with other theoretical estimations, while the ratios among the central values of the form-factors from different theoretical approaches vary in a large range and no definite conclusion can be made.

In the  $B$ -meson light-cone QCD sum rules for the  $B \rightarrow \pi, K, \rho, K^*, D, D^*, a_1(1260), K_0^*(1430), a_0(1450)$  form-factors [12, 13, 15, 16, 40], the main contributions come from the two-particle  $B$ -meson light-cone distributions  $\phi_{\pm}(\omega)$  as in the present case, the form-factors  $\propto \phi_{\pm}(\omega)$ , the model light-cone distribution  $\phi_+(\omega) = \frac{\omega}{\omega_0} \exp[-\frac{\omega}{\omega_0}]$  with the typical value  $\omega_0 = 0.46$  GeV can give satisfactory results. In the present case, the form-factors  $\propto \omega \phi_+(\omega)$  due to the derivative in the interpolating currents, if the minor contributions are neglected, the predictions based on the typical value  $\omega_0 = 0.46$  GeV are too large. There are two possible reasons to account for the apparent discrepancy: (1) the model light-cone distribution amplitudes  $\phi_{\pm}(\omega)$  are universal, but the  $B$ -meson light-cone QCD sum rules are not applicable for the  $B \rightarrow T$  form-factors, although we can prove that the operator product expansion near the light-cone  $x^2 \approx 0$  is feasible; (2) the  $B$ -meson light-cone QCD sum rules are applicable for the  $B \rightarrow T$  form-factors, but the model light-cone distribution amplitudes  $\phi_{\pm}(\omega)$  are not the ideal ones. A compromise between these two reasons can be suggested, both the model light-cone distribution amplitudes  $\phi_{\pm}(\omega)$  and the  $B$ -meson light-cone QCD sum rules are robust, we can search for the ideal value of the parameter  $\omega_0$ , as the line-shapes of the  $B$ -meson light-cone distribution amplitudes have significant impacts on the values of the form-factors.

The analytical expressions of the form-factors are complicated, we cannot obtain physical insight on the  $\omega_0$  dependence. We can neglect the tiny contributions from the three-particle  $B$ -meson light-cone distribution amplitudes and other minor contributions, recast the expressions into simple form, and study the  $\omega_0$  dependence. In the following, we will present some typical examples for illustration. The simplified expressions of the form-factors  $V_{B \rightarrow a_2}(0)$ ,  $A_{B \rightarrow a_1}(0)$  and  $F_{B \rightarrow \pi}^+(0)$  in

the transitions  $B \rightarrow a_2(1320)$ ,  $B \rightarrow a_1(1260)$  and  $B \rightarrow \pi$  can be written as

$$\begin{aligned} V_{B \rightarrow a_2}(0) &= \frac{f_B(m_B + m_{a_2})}{f_{a_2} m_{a_2}^2} e^{\frac{m_{a_2}^2}{M^2}} \int_0^{\frac{s_0}{m_B}} d\omega \omega \phi_+(\omega) e^{-\frac{\omega m_B}{M^2}}, \\ &= \frac{\omega_0 f_B(m_B + m_{a_2})}{f_{a_2} m_{a_2}^2} e^{\frac{m_{a_2}^2}{M^2}} \int_0^{\frac{s_0}{\omega_0 m_B}} dx x^2 e^{-(1 + \frac{\omega_0 m_B}{M^2})x}, \end{aligned} \quad (28)$$

$$\begin{aligned} A_{B \rightarrow a_1}(0) &= \frac{f_B(m_B - m_{a_1})}{2f_{a_1} m_{a_1}} e^{\frac{m_{a_1}^2}{M^2}} \int_0^{\frac{s_0}{m_B}} d\omega \phi_+(\omega) e^{-\frac{\omega m_B}{M^2}}, \\ &= \frac{f_B(m_B - m_{a_1})}{2f_{a_1} m_{a_1}} e^{\frac{m_{a_1}^2}{M^2}} \int_0^{\frac{s_0}{\omega_0 m_B}} dx x e^{-(1 + \frac{\omega_0 m_B}{M^2})x}, \end{aligned} \quad (29)$$

$$\begin{aligned} F_{B \rightarrow \pi}^+(0) &= \frac{f_B}{f_\pi} e^{\frac{m_\pi^2}{M^2}} \int_0^{\frac{s_0}{m_B}} d\omega \phi_-(\omega) e^{-\frac{\omega m_B}{M^2}}, \\ &= \frac{f_B}{f_\pi} e^{\frac{m_\pi^2}{M^2}} \int_0^{\frac{s_0}{\omega_0 m_B}} dx e^{-(1 + \frac{\omega_0 m_B}{M^2})x}, \end{aligned} \quad (30)$$

respectively, where the typical integral kernels are  $\omega \phi_+(\omega)$ ,  $\phi_+(\omega)$  and  $\phi_-(\omega)$ , respectively. We can take the input parameters as  $m_{a_2} = 1.31$  GeV,  $f_{a_2} = 0.107$  GeV,  $s_{a_2}^0 = 2.70$  GeV,  $M_{a_2}^2 = 1.2$  GeV<sup>2</sup> in the  $a_2(1320)$  channel,  $m_{a_1} = 1.23$  GeV,  $f_{a_1} = 0.238$  GeV,  $s_{a_1}^0 = 2.55$  GeV,  $M_{a_1}^2 = 1.2$  GeV<sup>2</sup> in the  $a_1(1260)$  channel and  $m_\pi = 0.14$  GeV,  $f_\pi = 0.13$  GeV,  $s_\pi^0 = 0.70$  GeV,  $M_\pi^2 = 1.0$  GeV<sup>2</sup> in the  $\pi$  channel, respectively [12, 13, 15], and plot the numerical results in Fig.3. From the figure, we can see that the values of the form-factors decrease monotonously with the increase of the  $\omega_0$ , in the region below the typical value  $\omega_0 = 0.46$  GeV [22], the form-factors decrease drastically, the curves are very steep, while in the region above the typical value  $\omega_0 = 0.46$  GeV, the form-factors decrease more slowly, and the curves are flatter. If we take the typical value  $\omega_0 = 0.46$  GeV, the values of the form-factors  $A_{B \rightarrow a_1}(0)$  and  $F_{B \rightarrow \pi}^+(0)$  are consistent with other theoretical estimations [12, 13, 15], while the value of the form-factor  $V_{B \rightarrow a_2}(0)$  is too large, on the other hand, if we take the value  $\omega_0 = 2\tilde{\lambda}_B = 0.92$  GeV, the value of the form-factor  $V_{B \rightarrow a_2}(0)$  is consistent with other theoretical estimations while the values of the form-factors  $A_{B \rightarrow a_1}(0)$  and  $F_{B \rightarrow \pi}^+(0)$  are too small.

Irrespective of either of the two possible explanations for the apparent discrepancy between the present theoretical calculation and the experimental extraction, we should bear in mind that the values of the parameter  $\omega_0$  from different theoretical approaches differ from each other greatly,  $\omega_0 = (0.35 \pm 0.15)$  GeV from the experiential value in the QCD factorization [17],  $\omega_0 = 0.37$  GeV,  $0.46 \pm 0.11$  GeV,  $0.6$  GeV from different QCD sum rules [19, 22, 41],  $\omega_0 = 0.7$  GeV from the Bakamjian-Thomas relativistic quark model [42],  $\omega_0 = (0.48 \pm 0.05)$  GeV from the operator product expansion [24]. The two form-factors in the decays  $B^- \rightarrow \gamma \ell^- \nu_\ell$  are proportional to  $\frac{1}{\lambda_B}$  at the tree-level in the heavy quark limit, those processes are directly related to the parameter  $\lambda_B = \omega_0$  and would be the most direct way of measuring it. In searching for the decays  $B^+ \rightarrow \gamma \ell^+ \nu_\ell$ ,  $\ell = e, \mu$ , the Babar collaboration has set the upper bounds  $\omega_0 > 0.67$  GeV or  $\omega_0 > 0.59$  GeV [43, 42],  $\omega_0 > 0.3$  GeV [44]. It is difficult to choose the ideal value at the present time. All those theoretical calculations and experimental extractions concern approximations in one or the other ways, and comprehensive theoretical analysis are still needed.

We can extract those form-factors from the precise experimental data on the radiative and semi-leptonic decays  $B \rightarrow T\gamma$ ,  $T\ell\bar{\ell}$  at the KEK-B and LHCb in the future by including the non-factorizable contributions, new physics effects, etc, in the theoretical analysis, and obtain severe constraints on the input parameter  $\omega_0$  of the  $B$ -meson light-cone distribution amplitudes, although it is a hard work. For example, the semi-leptonic decays  $B \rightarrow a_2(1320)l\nu_l$  and  $B \rightarrow f_2(1270)l\nu_l$  can be described by the effective Hamiltonian at the lowest order approximation in the standard model,

$$\mathcal{H}_{\text{eff}}(b \rightarrow ul\bar{\nu}_l) = \frac{G_F}{\sqrt{2}} V_{ub} \bar{u} \gamma_\mu (1 - \gamma_5) b \bar{l} \gamma^\mu (1 - \gamma_5) \nu_l, \quad (31)$$



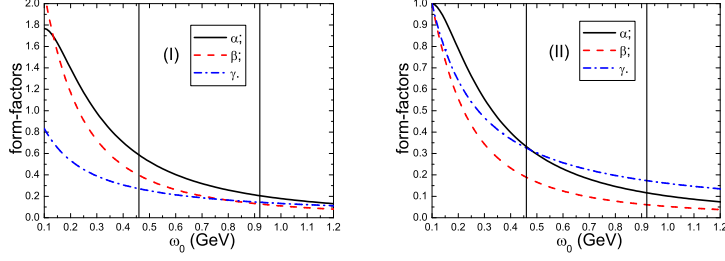


Figure 3: The form-factors with variation of the parameter  $\omega_0$  at zero momentum transfer, the  $\alpha$ ,  $\beta$  and  $\gamma$  denote the form-factors  $V_{B \rightarrow a_2}(0)$ ,  $A_{B \rightarrow a_1}(0)$  and  $F_{B \rightarrow \pi}^+(0)$ , respectively, the two vertical lines correspond to the typical values  $\omega_0 = 0.46$  GeV and  $0.92$  GeV, respectively. In (II), the form-factors are normalized to 1 at the value  $\omega_0 = 0.1$  GeV.

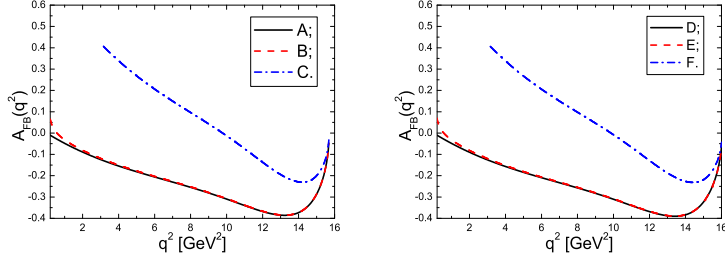


Figure 4: The forward-backward asymmetries  $A_{FB}(q^2)$  with the momentum transfer  $q^2$ , the  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $E$ , and  $F$  correspond to the decays  $\bar{B}^0 \rightarrow a_2^+(1320)e\bar{\nu}_e$ ,  $\bar{B}^0 \rightarrow a_2^+(1320)\mu\bar{\nu}_\mu$ ,  $\bar{B}^0 \rightarrow a_2^+(1320)\tau\bar{\nu}_\tau$ ,  $B^- \rightarrow f_2^0(1270)e\bar{\nu}_e$ ,  $B^- \rightarrow f_2^0(1270)\mu\bar{\nu}_\mu$ , and  $B^- \rightarrow f_2^0(1270)\tau\bar{\nu}_\tau$ , respectively.

where the  $V_{ub}$  is the CKM matrix element and the  $G_F$  is the Fermi constant. We can take the form-factors presented in Table 1 with  $\omega_0 = \lambda_B = 2\lambda_B$  as the input parameters to study the partial (and total) decay widths  $d\Gamma/dq^2$  (and  $\int_{m_l^2}^{(m_B - m_T)^2} dq^2 (d\Gamma/dq^2)$ ), and the forward-backward (FB) asymmetries  $A_{FB}$  of the lepton,

$$A_{FB}(q^2) = \frac{\int_0^1 dz (d\Gamma/dq^2 dz) - \int_{-1}^0 dz (d\Gamma/dq^2 dz)}{\int_0^1 dz (d\Gamma/dq^2 dz) + \int_{-1}^0 dz (d\Gamma/dq^2 dz)}, \quad (32)$$

where  $z = \cos \theta$  and the  $\theta$  is the polar angle of the lepton with respect to the moving direction of the tensor meson in the lepton pair rest frame. Taking the other parameters from the Review of Particle Physics [45], we can obtain the branching ratios  $1.6 \times 10^{-4}$ ,  $1.6 \times 10^{-4}$ ,  $0.6 \times 10^{-4}$ ,  $0.85 \times 10^{-4}$ ,  $0.85 \times 10^{-4}$  and  $0.34 \times 10^{-4}$  for the semi-leptonic decays  $\bar{B}^0 \rightarrow a_2^+(1320)e\bar{\nu}_e$ ,  $\bar{B}^0 \rightarrow a_2^+(1320)\mu\bar{\nu}_\mu$ ,  $\bar{B}^0 \rightarrow a_2^+(1320)\tau\bar{\nu}_\tau$ ,  $B^- \rightarrow f_2^0(1270)e\bar{\nu}_e$ ,  $B^- \rightarrow f_2^0(1270)\mu\bar{\nu}_\mu$  and  $B^- \rightarrow f_2^0(1270)\tau\bar{\nu}_\tau$ , respectively, which are consistent with the estimations from the perturbative QCD [11]. The forward-backward asymmetries  $A_{FB}(q^2)$  are shown in Fig.4. In this article, we show the central values explicitly.

## 4 Conclusion

In this article, we study the  $B \rightarrow K_2^*(1430), a_2(1320), f_2(1270)$  form-factors  $V(q^2)$ ,  $A_1(q^2)$ ,  $A_2(q^2)$ ,  $A_3(q^2)$ ,  $T_1(q^2)$ ,  $T_2(q^2)$  and  $T_3(q^2)$  with the  $B$ -meson light-cone QCD sum rules. In calculations, we observe that the dominating contributions come from the two-particle  $B$ -meson light-cone

distribution amplitude  $\phi_+(\omega)$ , its line-shapes have significant impacts on the values of the form-factors, we can search for the ideal values of the parameter  $\omega_0$ . In the  $B$ -meson light-cone sum rules for the  $B \rightarrow P, V, A, S$  form-factors, the dominating contributions  $\propto \phi_\pm(\omega)$ , while in the  $B$ -meson light-cone sum rules for the  $B \rightarrow T$  form-factors, the dominating contributions  $\propto \omega \phi_+(\omega)$ . If we take the value  $\omega_0 = \lambda_B = \tilde{\lambda}_B$  as in the  $B$ -meson light-cone QCD sum rules for the  $B \rightarrow P, V, A, S$  form-factors, the central values of the present predictions are at least (or almost) twice as large as the existing theoretical estimations, and the  $T_1(0)$  deviates greatly from the value extracted from the radiative decays  $B^+ \rightarrow K_2^{*+}(1430)\gamma$  and  $B^0 \rightarrow K_2^{*0}(1430)\gamma$ , the non-factorizable contributions are neglected in the extraction. On the other hand, if we take the value  $\omega_0 = \lambda_B = 2\tilde{\lambda}_B$ , the present predictions are compatible with other theoretical estimations. In calculations, we observe that the main uncertainty comes from the parameter  $\omega_0$  (or  $\lambda_B$ ), which determines the line-shapes of the two-particle and three-particle  $B$ -meson light-cone distribution amplitudes, it is of great importance to refine this parameter. We can extract the values of those form-factors from the experimental data on the radiative and semi-leptonic decays at the KEK-B and the LHCb in the future, and obtain severe constraints on the parameter  $\omega_0$ .

## Acknowledgments

This work is supported by National Natural Science Foundation of China, Grant Number 11075053, and the Fundamental Research Funds for the Central Universities.

## References

- [1] S. Nishida et al, Phys. Rev. Lett. **89** (2002) 231801.
- [2] B. Aubert et al, Phys. Rev. **D70** (2004) 091105.
- [3] H. Y. Cheng and K. C. Yang, Phys. Rev. **D83** (2011) 034001.
- [4] N. Isgur, D. Scora, B. Grinstein, and M. B. Wise, Phys. Rev. **D39** (1989) 799.
- [5] D. Scora and N. Isgur, Phys. Rev. **D52** (1995) 2783.
- [6] H. Y. Cheng, C. K. Chua and C. W. Hwang, Phys. Rev. **D69** (2004) 074025.
- [7] K. C. Yang, Phys. Lett. **B695** (2011) 444.
- [8] J. Charles, A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, Phys. Lett. **B451** (1999) 187.
- [9] D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. **D64** (2001) 094022.
- [10] A. Datta, Y. Gao, A. V. Gritsan, D. London, M. Nagashima and A. Szynekman, Phys. Rev. **D77** (2008) 114025.
- [11] W. Wang, Phys. Rev. **D83** (2011) 014008.
- [12] A. Khodjamirian, T. Mannel and N. Offen, Phys. Lett. **B620** (2005) 52.
- [13] A. Khodjamirian, T. Mannel and N. Offen, Phys. Rev. **D75** (2007) 054013.
- [14] H. Kawamura, J. Kodaira, C. F. Qiao and K. Tanaka, Phys. Lett. **B523** (2001) 111.
- [15] Z. G. Wang, Phys. Lett. **B666** (2008) 477.
- [16] S. Faller, A. Khodjamirian, C. Klein and T. Mannel, Eur. Phys. J. **C60** (2009) 603.

- [17] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. **B606** (2001) 245.
- [18] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. **B591** (2000) 313.
- [19] A. G. Grozin and M. Neubert, Phys. Rev. **D55** (1997) 272.
- [20] H. Kawamura, J. Kodaira, C. F. Qiao and K. Tanaka, Mod. Phys. Lett. **A18** (2003) 799.
- [21] B. O. Lange and M. Neubert, Phys. Rev. Lett. **91** (2003) 102001.
- [22] V. M. Braun, D. Yu. Ivanov and G. P. Korchemsky, Phys. Rev. **D69** (2004) 034014.
- [23] A. G. Grozin, Int. J. Mod. Phys. **A20** (2005) 7451.
- [24] S. J. Lee and M. Neubert, Phys. Rev. **D72** (2005) 094028.
- [25] S. Descotes-Genon and N. Offen, JHEP **0905** (2009) 091.
- [26] H. Hatanaka and K. C. Yang, Phys. Rev. **D79** (2009) 114008.
- [27] H. Hatanaka and K. C. Yang, Eur. Phys. J. **C67** (2010) 149.
- [28] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. **B147** (1979) 385.
- [29] L. J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rept. **127** (1985) 1.
- [30] V. M. Belyaev, V. M. Braun, A. Khodjamirian and R. Rückl, Phys. Rev. **D51** (1995) 6177.
- [31] H. n. Li and H. S. Liao, Phys. Rev. **D70** (2004) 074030.
- [32] T. Huang, C. F. Qiao and X. G. Wu, Phys. Rev. **D73** (2006) 074004.
- [33] P. Colangelo and A. Khodjamirian, hep-ph/0010175.
- [34] H. Y. Cheng, Y. Koike and K. C. Yang, Phys. Rev. **D82** (2010) 054019.
- [35] T. M. Aliev, K. Azizi and V. Bashiry, J. Phys. **G37** (2010) 025001.
- [36] A. Ali and A. Y. Parkhomenko, Eur. Phys. J. **C23** (2002) 89.
- [37] A. Khodjamirian, T. Mannel, A. A. Pivovarov and Y. M. Wang, JHEP **1009** (2010) 089.
- [38] A. S. Safir, Eur. Phys. J. direct **C3** (2001) 15.
- [39] H. Y. Cheng and C. K. Chua, Phys. Rev. **D81** (2010) 114006.
- [40] Z. G. Wang and J. F. Li, arXiv:1012.1704.
- [41] P. Ball and E. Kou, JHEP **0304** (2003) 029.
- [42] A. L. Yaouanc, L. Oliver and J. C. Raynal, Phys. Rev. **D77** (2008) 034005.
- [43] B. Aubert et al, arXiv:0704.1478.
- [44] B. Aubert et al, Phys. Rev. **D80** (2009) 111105.
- [45] K. Nakamura et al, J. Phys. **G37** (2010) 075021.